EMI

Faraday's Laws of Electromagnetic Induction:

First Law: Whenever there is a change in the magnetic flux in a closed circuit, an induced emf is produced

Second Law: The magnitude of the induced emf is directly proportional to the rate of change of magnetic flux linked with the circuit.

Φ : Magnetic flux linked with the coil

$|e| \ \alpha \ \frac{d\Phi}{dt}$

$If \ all \ units \ are \ in \ same \ system \ then \ |e| = \frac{d\Phi}{dt}$

$|e| = n\frac{d\Phi}{dt}, where \ n \ is \ the \ number \ of \ turns$

This is called flux rule.

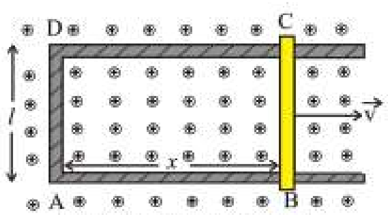
Lenz's Law:

The direction of induced current in a circuit is such that the magnetic field produced by the induced current opposes the change in the magnetic flux that induces it.

$e = -\frac{d\Phi}{dt} \ and \ for \ n \ turns \ e = -n\frac{d\Phi}{dt}$

NOTE: Lenz's Law follows directly from the conservation of energy.

Prove e = Blv for Translational motion of a conductor in a magnetic field:



⊕ Magnetic field $\vec{B}$ into the paper

A rectangular frame ABCD of area l.x is situated in a constant uniform magnetic field $\vec{B}$. As CB, of length l, moved outwards with a velocity v, the area of ABCD would increase. Thus, the flux through the loop increases with time, thus inducing a current from B to C.

$|e| = \frac{d\Phi}{dt} = \frac{d}{dt}(B.A) = \frac{d}{dt}(B.l.x) = B.l.\frac{dx}{dt} = B.l.v$

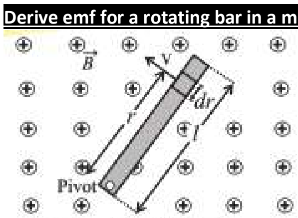
Using Lorentz force for the charge moving along BC

F = qBvsinθ=qBv where θ = 90° (angle between B and v)

$EMF \ e = \frac{W}{q} = F.l = B.v.l$

which is same as the above result.

Derive emf for a rotating bar in a magnetic field:



⊕ Magnetic field $\vec{B}$ into the paper

Consider a small segment dr of the bar at a distance r from the pivot, which is moving with velocity $\vec{v}$, in a magnetic field $\vec{B}$ and has an induced emf generated in it given by

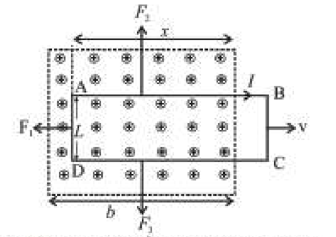
$de = B \ v \ dr$

Total induced emf $e = \int de = \int Bvdr$

$$e = \int B\omega r dr = B\omega\int_0^l r dr = B\omega\frac{l^2}{2}$$

$$e = \frac{1}{2}B\omega l^2$$

Energy/Heat transfer in Translatory Motion:



⊕ Magnetic field $\vec{B}$ into plane of the paper

Consider loop ABCD moving with constant velocity $\vec{v}$, in a uniform magnetic field B as shown. The induced current is as shown and the forces too as shown. F_2 and F_3 will cancel out each other. Let $\vec{F}$ be the applied force.

As the loop is moved to the right, the area within the magnetic field decreases, flux linked with the coil decreases, which induces a current in the loop, which creates a force in the opposite direction (i.e. F_1).

As we know |e| = BLv

$i = \frac{|e|}{R} = \frac{BLv}{R}$

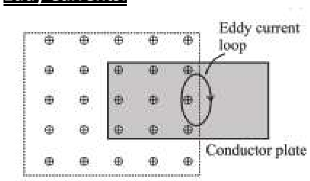
$|F| = |F_1| = BIL = B.\ \frac{BLv}{R}.L = \frac{B^2L^2v}{R}$

$Mechanical \ P = F.v = \frac{B^2L^2v}{R}.v = \frac{B^2L^2v^2}{R}$

$Electrical \ P = i^2R = \left(\frac{BLv}{R}\right)^2.R = \frac{B^2L^2v^2}{R}$

Thus, work done in pulling a loop through a magnetic field appears as heat energy in that loop.

Eddy Currents:



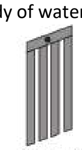
⊕ Magnetic field into the paper

Suppose a metal plate is moved relative to the magnetic field, then there will be induced current (due to change of flux). This induced current will swirl about within the plate (as if they were caught in an eddy of water).

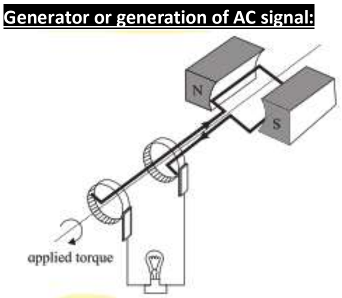
Such currents are called eddy currents.

Drawback: They are responsible for dissipation of energy as heat energy.

Remedy: Eddy current can be reduced by discontinuity in structure of the conducting plate


Reduction in eddy currents

Generator or generation of AC signal:



Conversion:
Mechanical Energy to Electrical Energy

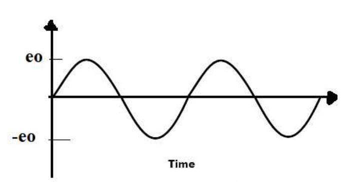
The armature is rotated by some external agency. This causes the wire frame to cut the magnetic lines of force, thus inducing an emf, which is proportional to the speed of rotation ω.

$\Phi = \bar{B}.\bar{A} = BAcos\theta = BAcos(\omega t)$

let N be the number of turns

$e = -N\frac{d\Phi}{dt} = -N\ \frac{d}{dt}(BAcos\omega t) = NAB\omega\sin(\omega t) = e_o\sin(\omega t)$

Where e_0 = NABω and ω=2πf where f : frequency of revolution of coil



Self Inductance:

Consider a coil with changing current. The changing current will create a varying magnetic flux linked with the same coil, and this varying magnetic flux will create induced emf in the same coil. This is termed as self-inductance.

$\emptyset \alpha I$

$\emptyset = LI$, where L: self-inductance or coefficient of self induction of the coil.

It depends on geometry and material properties of the coil

SI unit of self-inductance: Henry (H)

$$e = -\frac{d\phi}{dt} = -\frac{d}{dt}(LI) = -L\frac{dI}{dt}$$

$$|e| = L\frac{dI}{dt}$$

Definition of self inductance: It is equal to the induced emf produced per unit rate of change of current in it.

NOTE: for series Inductors $L=L_1+L_2+\ldots$.

And for parallel inductors $1/L = 1/L_1 + 1/L_2 + \ldots\ldots$.

Energy stored in a magnetic field/work done to establish a steady current/work done in moving a charge against an induced emf:

Consider a coil with changing current. The changing current will create a varying magnetic flux linked within the same coil, and this varying magnetic flux will create induced emf. To establish a growing magnetic field, work has to be done against this induced emf. This work or energy spent is the cause of heat in the circuit.

$$dW = -e.dq = -e.I.dt = L\frac{dI}{dt}.I.dt = LIdI$$

Thus,total work done $W = U = \int_0^I LIdt = L\int_0^I Idt = \frac{1}{2}LI^2$

Inductance of a solenoid:

Consider N coils of a solenoid of length l, such that n=N/l where n is turns per unit length. Let i be the current flowing in the windings and $\emptyset$ is the flux through the central region of the solenoid. Let B be the magnetic field and A is the cross-sectional area of the solenoid.

$$\phi = \overline{B}.\overline{A} = BA\cos\theta = BA,\ Since\ \theta = 0^0$$

$$L = \frac{N\phi}{i} = \frac{(nl)BA}{i} = \frac{nl(\mu_o ni)A}{i} = \mu_o n^2 lA\,,$$

where l.A is the internal volume of the solenoid

$$\frac{L}{l} = \mu_o n^2 A = \mu_o n^2\left(\frac{\pi d^2}{4}\right)$$ Where d : diameter of solenoid

$$\boldsymbol{Thus,\ L\alpha\ n^2\ and\ L\alpha\ d^2}$$

$$Note:\ Energy\ density = \frac{energy}{volume} = \frac{\frac{1}{2}LI^2}{Al} = \left(\frac{L}{l}\right)\frac{I^2}{2A}$$
$$= \mu_o n^2 A\frac{I^2}{2A} = \frac{1}{2}\mu_o n^2 I^2 = \frac{B^2}{2\mu_o}$$

Mutual Inductance:

If coil1 has current I1. This will create a magnetic field B1 in the vicinity of coil1. Let $\emptyset 21$ be the magnetic flux linked with coil2. Then,

$$\phi_{21}\alpha\, I_1$$

$\phi_{21} = M_{21}I_1$,where M_{21} : coefficient of mutual induction or mutual inductance of coil2 with respect to coil1.

If I_1 changes then $\emptyset_1$ changes, which links with coil2, thus inducing a changing $\emptyset_{21}$, which induces a changing I_2 and hence induces an emf

$$e_{21} = -\frac{d\phi_{21}}{dt} = -M_{21}\frac{dI_1}{dt}$$

this changing I_2 will create a changing flux $\emptyset_2$ associated with coil2, which will link back to coil1 as $\emptyset_{12}$

$$\phi_{12}\alpha\, I_2$$

$\phi_{12} = M_{12}I_2$, where M_{12}: coefficient of mutual induction or mutual inductance of coil1 with respect to coil2.

And this changing flux $\emptyset_{12}$ induces and emf

$$e_{12} = -\frac{d\phi_{12}}{dt} = -M_{12}\frac{dI_2}{dt}.$$

By symmetry $M_{12}= M_{21}=M$

Hence ***mutual inductance is defined as*** flux linked with one circuit per unit current in the other circuit.

We also define coil1 as primary (input) and coil2 as secondary (output).

$$e_s = -\frac{d\phi_s}{dt} = -\frac{d}{dt}(MI_p) = -M\frac{dI_p}{dt}$$

Hence, **mutual inductance can also be defined as** emf induced in the secondary per unit rate of change of primary current

SI unit of mutual inductance: Henry (H)

Mutual inductance between two circuits is 1H if 1V is induced in the secondary for 1A/s rate of change of primary current.

Coefficient of Coupling:

The amount of flux linkage from coil1 to coil2 decides the coefficient of coupling(K) and hence the Mutual induction between the coils.

$$e_{21}\alpha\, M$$
$$e_{21}\alpha\, N_2 d\phi_2$$
$$\alpha\, N_2 K d\phi_1$$
$$\alpha\, N_2 K N_1$$
$$Thus,\ M = KN_1N_2\quad But,\ L\alpha N^2$$
$$Thus,\ M = K\sqrt{L_1L_2}$$

K = 1 : Perfect coupling and if L1 = L2 =L then M=L

K > 0.5 : tightly coupled

K < 0.5 loosely coupled

for K = 1, the two coils are wound on a common iron core.

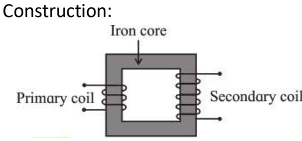
If two separate cores or air cores are used then K depends on distance between the coils and the angle (parallel coils K will be maximum and perpendicular coils K is minimum)

Transformer:

Purpose: Change voltage from high value to low value or vice versa.

Principle: Mutual Induction

Construction:



It consists of two coils, primary and secondary, insulated from each other and wrapped on a common soft iron core.

Working: When an AC voltage is applied at the primary, the current in the primary changes and this creates a changing magnetic flux in the core, which links with the secondary, thus inducing an alternating emf in the secondary.

If ϕ is the flux linked per turn at any instant t, and Ns and Np are the number of turn of secondary and primary coil, then

$\phi_P = N_P\phi$ and $\phi_S = N_S\phi$

This changing flux will induce an emf in both coils

$$e_P = -\frac{d\phi_P}{dt} = -N_P\frac{d\phi}{dt}\quad and\ e_S = -\frac{d\phi_S}{dt} = -N_S\frac{d\phi}{dt}$$

$$Thus, \frac{e_S}{e_P} = \frac{N_S}{N_P}, where\ \frac{N_S}{N_P} = turns\ ratio$$

$$If\ transformer\ is\ 100\%\ efficient,$$
$$Input\ power\ = output\ power$$

$$e_P i_P = e_S i_S, thus\ \frac{e_s}{e_p} = \frac{i_p}{i_s}.\ Hence, \frac{e_S}{e_P} = \frac{N_S}{N_P} = \frac{i_p}{i_s}$$

Case 1: Step up transformer

$N_S > N_P$, hence $e_S>e_P$ and $i_P<i_S$

Case 2: Step down transformer

$N_S < N_P$, hence $e_S<e_P$ and $i_P>i_S$